GOVT. M.H.COLLEGE OF HOME SC. & SCIENCE FOR WOMEN AUTONOMOUS
JABALPUR (M.P.)

Code No. : OBS/201/M/21

M.SC. II SEMESTER MAIN EXAMINATION JULY - 2021
Subject : Mathematics
Paper - I
Advanced Abstract Algebra - II

Max. Marks : 35
Min. Marks : 12

Note : Attempt all questions. All questions carry equal marks. Symbols have their usual meanings.

UNIT - I

Q.1.   Let K be a normal extension of F and let H be a subgroup of G(K, F) : let   7

$K_H = \{x \in K : \sigma(x) = x \text{ for all } \sigma \in H\}$

be the fixed field of H. Then prove that

1. $[K : K_H] = 0 (H)$

2. $H = G(K, K_H)$

In particular, when H = G(K, F)

$$[K : F] = 0 (G(K, F))$$

UNIT - II

Q.2.   State and prove fundamental theorem of R-homomorphism.   7

UNIT - III

Q.3.   Let $M = \Sigma_{\alpha \in \wedge} M_\alpha$ be a sum of simple R-submodules M_α. Let K be a submodule of M. Then

prove that there exists a subset $\wedge'$ of $\wedge$ such that $X = \Sigma_{\alpha \in \wedge'} M_\alpha$ is a direct sum and

$M = K \oplus \left(\oplus \Sigma_{\alpha \in \wedge'} M_\alpha \right)$

UNIT - IV

Q.4.   State and prove Hilbert basis theorem.

UNIT - V

Q.5.   Find the abelian group generated by (x_1, x_2, x_3) subject to

$$5x_1 + 9x_2 + 5x_3 = 0$$
$$2x_1 + 4x_2 + 2x_3 = 0$$
$$x_1 + x_2 - 3x_3 = 0$$

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GOVT. M.H.COLLEGE OF HOME SC. & SCIENCE FOR WOMEN AUTONOMOUS JABALPUR (M.P.)

Code No. : OBS/202/M/21

M.SC. II SEMESTER MAIN EXAMINATION JULY - 2021
Subject : Mathematics
Paper - II
Advanced Discrete Mathematics

Max. Marks : 35
Min. Marks : 12

Note : Attempt all questions. All questions carry equal marks. Symbols have their usual meanings.

UNIT - I

Q.1. Prove that every finite semigroup has an idempotent element. **7**

UNIT - II

Q.2. Show that dual of a lattice is a lattice. **7**

UNIT - III

Q.3. Explain direct product of lattice. Prove that direct product of lattice is a lattice. **7**

UNIT - IV

7

Q.4. Explain with examples -

(a) (i) walk   (ii) path   (iii) circuit.

(b) Prove that number of odd vertices in a graph is always even.

UNIT - V

Q.5. (a) Define tree & explain its properties. **3½**

(b) Prove that tree T with n vertices has n-1 edges. **3½**

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GOVT. M.H.COLLEGE OF HOME SC. & SCIENCE FOR WOMEN AUTONOMOUS JABALPUR (M.P.)

Code No. : OBS/203/M/21

M.SC. II SEMESTER MAIN EXAMINATION JULY - 2021
Subject : Mathematics
Paper - III
Complex Analysis - II

Max. Marks : 35
Min. Marks : 12

Note : Attempt all questions. All questions carry equal marks. Symbols have their usual meanings.

UNIT - I

Q.1.   State and prove Weierstrass factorization theorem.   **7**

UNIT - I I

Q.2.   State and prove Schwartz reflection principle.   **7**

UNIT - III

Q.3.   Let (f, θ) be a function element and let G be a region containing D such that (f, θ) admits unrestricted continuation in G. Let $a \in D$, $b \in G$ and Let Y_0 and Y_1, be paths in G from a to b, Let $\{(f_t, D_t) : 0 \le t \le 1\}$ and $\{(g_t, D_t) : 0 \le t \le 1\}$ be analytic continuation of (f, D) along Y_0 and Y_1 respectively. If Y_0 and Y_0 are fixed-end-point homotopic in G then prove that $[f_1]_{-b} = [g_1]_b$

UNIT - IV

Q.4.   State and prove Poision-Jenson Formula.

UNIT - I

Q.5.   Let f be an analytic function in a region containing the closure of the disc $D = \{Z : |Z| < 1\}$ and satisfying $f(0) = 0$, $f'(0) = 1$. Then there is a disc $S \subset D$ in which f is one-one and such that $f(S)$ contains a disc of radius 1/72.

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Govt. M.H.College of Home Sc. & Science for Women Autonomous Jabalpur (M.P.)

Code No. : OBS/204/M/21

M.Sc. II Semester Main Examination July - 2021
Subject : Mathematics
Paper - IV
Lebesgue Measure & Integration

Max. Marks : 35
Min. Marks : 12

Note : Attempt all questions. All questions carry equal marks. Symbols have their usual meanings.

UNIT - I

Q.1.(a) Prove that the outer measure of an interval equals its length. $3\frac{1}{2}$

(b) Prove that the class M is a σ - algebra. $3\frac{1}{2}$

UNIT - I I

Q.2. (a) State & prove the lebesgue monotone convergence theorem. $3\frac{1}{2}$

(b) Show that $\int_0^1 \frac{x^{\frac{1}{3}}}{1-x} \log\frac{1}{x} dx = 9\sum_{n=1}^{\infty} \frac{1}{(3n+1)^2}$ $3\frac{1}{2}$

UNIT - III

Q.3. (a) If $f \in$ BV [a, b] then show that f(b) - f(a) = P - N & T = P + N μ variations being on the finite interval [a, b] $3\frac{1}{2}$

(b) Prove that $f \in$ BV [a, b] , where a & b are finite if and only if the graph of f is rectifiable.

UNIT - IV

Q.4. (a) Let ψ be strictly convex then show that $\psi\left(\int d\mu\right) = \int (\psi_0 f) \ d\mu$ if and only if f = $\int$ f dμ a.e.

(b) Prove that every function convex on an open interval is continuous. $3\frac{1}{2}$

UNIT - V

Q.5. (a) If a sequence of measurable functions converges in measure, then prove that the limit function is unique a.e. $3\frac{1}{2}$

(b) If {fn}is a sequence of measurable function which is fundamental in measure then prove that there exists a measurable function f such that fn $\to$ f in measure. $3\frac{1}{2}$

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GOVT. M.H.COLLEGE OF HOME SC. & SCIENCE FOR WOMEN AUTONOMOUS
JABALPUR (M.P.)

Code No. : OBS/205/M/21

M.Sc. II SEMESTER MAIN EXAMINATION JULY - 2021
Subject : Mathematics
Paper - V
Ordinary & Partial Differential Equations

Max. Marks : 35
Min. Marks : 12

Note : Attempt all questions. All questions carry equal marks. Symbols have their usual meanings.

UNIT - I

Q.1.(a) Prove that the eigen functions of a strum Liouville problem form an orthogonal set. 3½

(b) Solve $x^2y'' - 2(x^2 + x) y' + (x^2 + 2x + 2) y = 0$ by reducing it to normal form. 3½

UNIT - II

Q.2. (a) Solve the equation $z (x + 2y) p - z (y + 2x)q = y^2 - x^2$ 3½

(b) Slove by Chrpit's method the equation $x z q^2 = p$ 3½

UNIT - III

Q.3. (a) Find solution of $y = (1 + x)^p$ by method of power series. 3½

(b) Solve the Legendre's equation by method of power series. 3½

UNIT - IV

Q.4. (a) Find $L(\sin^2 ax)$ without integrating. 3½

(b) Find the function whose Laplace transorm is $\dfrac{1}{p^4 + p^2}$ 3½

UNIT - V

Q.5. (a) Find $L^{-1}\left[\dfrac{1}{\left(p^2 + a^2\right)^2}\right]$ by convolution. 3½

(b) Solve the integral equation $y (x) = x^3 + \int_0^x \sin (x - t) y (t)\, dt$ 3½

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